TOPOLOGY I - MID-SEMESTRAL EXAM

Time : 2 hours Max. Marks : 40

Answer all questions. You may use results proved in class after correctly quoting them. Any other claim must be accompanied by a proof.

(1) Decide whether the following statements are *True* of *False*. Answers without correct and complete justifications will not be given any marks.
 (a) $[0,1] \times [0,1)$ is a linear continuum in the dictionary order.
 (b) The dictionary order topology and the product topology on $[0,1] \times [0,1]$ are identical.
 (c) If τ is the usual topology on $\mathbb{R}$, then there exists a topology $\tau' \subseteq \tau$ such that $(\mathbb{R}, \tau')$ and S^1 are homeomorphic.
 (d) If C is a connected subset of $\mathbb{R}^2$, then $\operatorname{Int} C$ is path connected. [4x3]

(2) Show that a function $f : X \longrightarrow Y$ between topological spaces is continuous if and only if for every subset $A \subseteq X$ we have $f(\bar{A}) = \bar{f(A)}$. [4]

(3) Let X be the quotient space obtained from $\mathbb{R} \times \{0,1\}$ by the equivalence relation generated by declaring
$$(x,0) \sim (x,1)$$
for every $x \in \mathbb{R}$ with $|x| > 1$.
 (a) Does X satisfy the T_1 axiom? Why or why not?
 (b) Is X Hausdorff? Why or why not? [4+4]

(4) Let X be the space
$$X = \{(x, x/n) : 0 \leq x \leq 1, n = 1, 2, \ldots\} \cup \{(x, 0) : 0 \leq x \leq 1\}$$
with the subspace topology of $\mathbb{R}^2$. Show that X is not locally path connected. [8]

(5) Let $\sim$ be the equivalence relation defined on S^1 by setting $x \sim y$ if and only if $x = \pm y$. Show that the quotient space $S^1/\sim$ is homeomorphic to S^1. [8]